OPTIMAL CONTROL OF PURE JUMP MARKOV PROCESSES WITH NOISE-FREE PARTIAL OBSERVATION

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In this talk I am going to address an optimal control problem with partial observation, mainly characterized by observations not corrupted by noise. Let X and Y be a given pair of stochastic processes, defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$, with values in two complete and separable metric spaces I and O respectively. The unobserved (or signal) process X is a time-homogeneous pure jump Markov process, whose rate transition measure is known. The observed process Y is defined as $Y_t = h(X_t)$, $t \geq 0$, where $h \colon I \to O$ is a known surjective and measurable function.

An optimal control problem with partial observation requires a two-step procedure. The first is related to the *filtering problem* (see [1]). I will provide an explicit SDE for the *filtering process* $(\pi_t)_{t \geq 0}$, satisfying

$$\pi_t(\varphi) = \mathbb{E}\big[\varphi(X_t) \mid \mathcal{Y}_t\big]$$

for all $t \geq 0$ and all $\varphi \colon I \to \mathbb{R}$ bounded and measurable functions; here $(\mathcal{Y}_t)_{t \geq 0}$ denotes the natural filtration of Y, i.e. $\mathcal{Y}_t = \sigma(Y_s \colon 0 \leq s \leq t)$, for all $t \geq 0$. The filtering process is also characterized as a *Piecewise Deterministic Markov Process* (see [2]).

The second step is the solution of the optimal control problem. The aim is to minimize a discounted cost functional on infinite time horizon by controlling the rate transition measure of the unobserved process via the information provided by the observed process. This can be studied thanks to the associated *separated problem*, that is a discrete-time optimal control problem for the filtering process with complete observation. I will show the equivalence between the original and the separated problems and prove a characterization of the value function of the latter problem as the unique fixed point of a suitably defined contraction mapping.

In the case of unobserved process given by a finite-state controlled Markov chain (i.e. when the space I is of finite cardinality), I will characterize the value function of the separated problem as the unique constrained viscosity solution (in the sense of Soner, see [3]) of an integro-differential HJB equation. Finally, I will show the existence of an optimal control. In this setting such an optimal control coincides with a *piecewise open-loop* control (see [4]).

References

[1] A. Bain and D. Crisan. *Fundamentals of Stochastic Filtering.* Springer, New York, 2009.

[2] M.H.A. Davis. *Markov Models and Optimization*, volume 49 of *Monographs on Statistics and Applied Probability.* Chapman and Hall, London, 1993.

[3] H. M. Soner. Optimal control with state-space constraint. II. *SIAM J. Control Optim.*, 24(6):1110–1122, 1986.

[4] D. Vermes. Optimal control of piecewise deterministic Markov process. *Stochastics*, 14(3):165–207, 1985.